SAMSKRUTI COLLEGE OF ENGINEERING & TECHNOLOGY

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Course Hand Out

Subject Name: COSM

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UNIT –I

KEY POINTS:

Definitions

Experiment: a set of conditions under which some variable is observed Outcome of an experiment: the result of the observation (a sample point) Sample Space(S): collection of all possible outcomes (sample points) of an experiment

Event: a collection of sample points

• Operations with eventsAc

1. Complementation AcA2. Intersection A BA∩B 3. Union A∪B BA

• Properties of events 1. Mutual Exclusiveness - intersection of events is the null set (Ai∩Aj = ∅, for all i ≠ j)

2. Collective Exhaustiveness (C.E.) - union of events is sample space (A1∪A2∪...∪An = S)

3. If the events {A1, A2, ... , An} are both mutually exclusive and collectively exhaustive, they form a partition of the sample space, S.

• Probability of events

• Relative frequency fE and limit of relative frequency FE of an event E fE =nEnFE =f lim E =lim nE n∞→n∞→n

• Properties of relative frequency (the same is true for the limit of relative frequency

1. 0 ≤ fE ≤ 1 2. fS = 1

3. f(A∪B) = fA + fB if A and B are mutually exclusive • Properties/axioms of probability

1. 0 ≤ P(A) ≤ 1

2. P(S) = 1 3. P(A∪B) = P(A) + P(B) if A and B are mutually exclusive

• Two consequences of the axioms of probability theory 1. P(Ac) = 1 − P(A) 2. P(A∪B) = P(A) + P(B) − P(A∩B), for any two events A and B, ⇒P(A∩B) = P(A) + P(B) - P(A∪B)

• Conditional Probability Definition: P(A | B)=P(A ∩B)P(B) Therefore, P(A∩B) can also be obtained as P(A∩B) = P(B)P(A|B) = P(A) P(B|A)

• Total Probability Theorem Let {B1, B2,..., Bn} be a set of mutually exclusive and collectively exhaustive events and let A be any other event. Then the marginal probability of A can be obtained as: P(A) =∑P(A ∩B) =∑P(B)P(A | B)i iiii

• Independent events A and B are independent if: P(A|B) = P(A), or equivalently if P(B|A) = P(B), or if P(A∩B) = P(A) P(B)

• Bayes' Theorem

| A)P(A | B) =P(A) P(B P(B) Using Total Probability Theorem, P(B) can be expressed in terms of P(A), P(Ac) = 1 – P(A), and the conditional probabilities P(B|A) and P(B|AC): P(B) =P(A)P(B | A) +P(AC)P(B | AC) So Bayes' Theorem can be rewritten as: P(A| B) =P(A) P(B| A) CP(A)P(B| A)+)P(B P(A | AC

Short Questions

1. Define sample space .Give one example
2. Define mutually exclusive events with an example
3. Define random variable
4. Derive mean of a binomial distribution
5. Prove that E(ax + b) = a E(x) + b
6. A random variable X has the following probability distribution

X	1	2	3	4	5	6	7	8
P(X)	k	2k	3k	4k	5k	6k	7k	8k

   Find K

7. A random variable X is defined as sum of the numbers on the faces when two dice are thrown. Find the mean of X
8. Find the constant K such that f(x)= $\begin{cases} kx^2 & if\ 0 < x < 3 \\ 0 & otherwise \end{cases}$
9. For the following probability distribution

X	-3	6	9
P(X)	1/6	1/2	1/3

Find E(X)

10. If X is normal variate find the area A

    i) To the left of $z = -1.78$

    ii) To the right of $z = -1.45$

    iii) Corresponding to $-0.8 \le z \le 1.53$

Long Questions

1. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6
P(X)	0	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find i) K ii) mean iii) variance

2. If the probability density of a random variable is given by f(x)= $\begin{cases} k\,(1-x^2\,) & if\;\; 0 < x < 1 \\ 0 & otherwise \end{cases}$

   Find the value of k and the probabilities that a random variable having the probability density will take on a value i) between 0.1 and 0.2 ii) greater than 0.5

3. Four coins are tossed 160 times. The number of times X heads occurs is given below

X	0	1	2	3	4
No. of times	8	34	69	43	6

4 a) Six dice are thrown 729 times. How many times do you expect at least 3 dice to show a 5 or 6?

b) Out of 800 families with 5 children each, how many would you expect to have i) 3 boys ii) 5 girls

5. If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kgs. How many have masses i) greater than 72 kgs ii) less than or equal to 64 kgs iii) between 65 and 71 kgs inclusive

6. 1000 students have written an examination, the mean of test is 35 and standard deviation is 5. Assuming the distribution to be normal, find i) how many students marks lie between 25 and 40 ii) how many students get more than 40 iii) how many students get less than 20

7. A population consists of the four numbers 1,5,6,8. Consider all samples of size 2 drawn without replacement from this population. Find

i) Population mean

ii) Population standard deviation

iii) Mean of sampling distribution of means

iv) Standard deviation of sampling distribution of means

8. a) Write the chief characteristics of normal distribution

b) If X is normal variate find the area A

i) To the left of $z = -1.78$

ii) To the right of $z = -1.45$

iii) Corresponding to $-0.8 \le z \le 1.53$

9. A population consists of 2,3,6,8,11. Consider all samples of size 2 drawn with replacement from this population. Find

i) Population mean

ii) Population standard deviation

iii) Mean of sampling distribution of means

iv) Standard deviation of sampling distribution of means

10. A random sample of size 64 is taken from a normal population with mean 51.4 and standard deviation 68. What is the probability that the mean of a sample will i) exceed 52.9 ii) fall between 50.5 and 52.3 iii) less than 50.6

Fill in the Blanks

1) The probability of getting 2 heads in tossing 5 coins ________5/256________

2) If a coin is tossed 6 times in succession, the probability of getting at least one head is 63/24

3) The mean of the binomial distribution is ________ np

4) If n and p are the parameters of a binomial distribution, the standard deviation of this distribution is _____npq

5) The probability of having at least one tail in four throws with a coin is ________15/26-------

6) If mean of the binomial distribution is 8 and Variance is 6, the mode of this distribution is -----8--

7) If mean of the binomial distribution is 6 and Variance is 2, then mode of this distribution is ----------------6-----------

8) The mean of binomial distribution is 4 and variance is 2 then p= --------1/2-------------------

9) The probability of getting one boy in a family of 4 children is ------1/4----------------

10) If the mean and variance of a binomial variate are 12 and 4, then the distribution is --------0.83

Multiple Choice Questions

1) If a coin tossed twice, the probability of getting at least one head is (C)

   a) 1/2 b)1/4 c)3/4 d)None

2) The probability of getting a number greater than 2 or an even number in a single thrown of a fair die is

   (B)

   a) 1/3 b) 2/3 c) 5/6 d)None

3) A bag contains 3 red balls, 4 White balls and 7 black balls. The probability of drawing a red or a black ball is

   (B)

   a) 2/7 b) 5/7 c) 3/7 d) 4/7

4) The odd in favor of drawing a king or a diamond from a well shuffled pack are (B)

   a) 9:4 b)4:9 c) 5:9 d) 9:5

5) The probability that a leap year will have 53 Tuesdays is (C)

   a)1/7 b) 3/7 c)2/7 d) 5/7

6) The Probability of at least one of the events A and B Occurs is 0.6. If A and B occurs

Simultaneously with probability 2 then P (A^1) + P (B^1) = (C)

a) 0.4 b) 1.2 c) 0.8 d) 1.4

b)

7) A and B^1 are two independent events such that $P(A^1 \wedge B) = 8/25$ and $P(A \wedge B^1) = 3/25$, then P(A) is

(D)

a) 2/5 b) 4/5 c) 1/5 d)3/5

8) For two events A and B if P(A) = P(A/B) =1/4 and P(B/A) =1/2, then (B)

(a) A and B are mutually exclusive

(b) A and B are independent

c) P (A^1/b) = 3/4 d) None

9) If a coin is tossed 6 times in succession, the probability of getting at least one head is (C)

a) 1/64 b)3/32 c) 63/64 d) None

10) A coin is tossed n times. The probability that the head will present itself an odd number of times is

(C)

a) $1/2^n$ b) 1/2n c)1/2 d)413